National University of Technology, Islamabad
Assignment I (Linear Algebra and ODE), Fall 2019
Due Date: October 08, 2019

Q.1 Give brief answers to the following questions.

(a) Give an example of two row equivalent matrices in echelon form. Note that although the reduced echelon form is unique, there will be several different row equivalent matrices in echelon form.

(b) Suppose a 4×6 coefficient matrix has 4 pivot columns. Is the corresponding system of equations consistent? Justify your answer.

(c) If a 7×5 augmented matrix has a pivot in every column, what can you say about the solution to the corresponding system of equations? Justify your answer.

(d) If a **consistent** system of equations has more unknowns than equations, what can be said about the number of solutions?

(e) Explain the concept of elementary matrices? Give three examples of elementary matrices obtained by different elementary row operations.

Q.2 Find the values of k for which the system of equations $\begin{cases} x + ky = 1 \\ kx + y = 1 \end{cases}$ has no solution, exactly one solution and infinitely many solutions.

Q.3 Consider the following homogeneous system of linear equations (where a and b are non-zero constants)

$$\begin{aligned} x + 2y \quad\quad &= 0 \\ ax + 8y + 3z &= 0 \\ by + 5z &= 0. \end{aligned}$$

(a) Find a value for a which will make it necessary during Gaussian elimination to interchange rows in the coefficient matrix.

(b) Suppose that a is different than the value you found in part (a). Find a value for b (of course in terms of a) so that the system has a non-trivial solution.

(c) Suppose that a is different than the value you found in part (a) and that $b = 100$. Suppose further that a is chosen so that the solution to the system is **not** unique. For what values of α and β, the general solution to the system (in terms of the free variable z) is $\left(\frac{z}{\alpha}, \frac{z}{\beta}, z\right)$?

(d) Write down the solution set obtained in the part (c) as a **span** of a set of vectors.

(e) Give a geometric interpretation of the solution set obtained in the part (c).

Q.4 Let $\mathbf{w} = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}^T$, $\mathbf{x} = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^T$, $\mathbf{y} = \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}^T$, and $\mathbf{z} = \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^T$.

(a) We can show that $\{\mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is not a spanning set for $\mathbb{R}^4$ by finding a vector $\mathbf{u}$ in $\mathbb{R}^4$ such that $\mathbf{u} \notin \text{span}\{\mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}$. Find unknown a such that the vector $\mathbf{u} = \begin{bmatrix} 1 & 2 & 3 & a \end{bmatrix}$ is not in $\text{span}\{\mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}$.

(b) Show that $\{\mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a linearly dependent set by writing $\mathbf{z}$ as a linear combination of vectors $\mathbf{w}$, $\mathbf{x}$, and $\mathbf{y}$.

"Your problem isn't the problem, its your attitude about the problem." — Ann Brashares.