National University of Technology, Islamabad
Assignment III (Calculus II), Spring 2019
Solution Key

Q1. **(a)** Along $y = 0$: $\lim\limits_{x\to 0} \dfrac{x}{x^2} = \lim\limits_{x\to 0} \dfrac{1}{x}$ does not exist, so the original limit does not exist.

(b) Along $y = 0$: $\lim\limits_{x\to 0} \dfrac{1}{x^2}$ does not exist, so the original limit does not exist.

Q2. **(a)** Along $y = mx$: $\lim\limits_{x\to 0} \dfrac{mx^4}{2x^6 + m^2x^2} = \lim\limits_{x\to 0} \dfrac{mx^2}{2x^4 + m^2} = 0$; along $y = kx^2$: $\lim\limits_{x\to 0} \dfrac{kx^5}{2x^6 + k^2x^4} = \lim\limits_{x\to 0} \dfrac{kx}{2x^2 + k^2} = 0$.

(b) $\lim\limits_{x\to 0} \dfrac{x^6}{2x^6 + x^6} = \lim\limits_{x\to 0} \dfrac{1}{3} = \dfrac{1}{3} \neq 0$.

Q3. **(a)** $P = 10T/V$, $\partial P/\partial T = 10/V$, $\partial P/\partial T|_{T=80,\ V=50} = 1/5$ lb/(in^{2}K).

(b) $V = 10T/P, \partial V/\partial P = -10T/P^2$, if $V = 50$ and $T = 80$, then $P = 10(80)/(50) = 16$, $\partial V/\partial P|_{T=80,\ P=16} = -25/8$(in^5/lb).

Q4. $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\dfrac{dy}{dt} + \dfrac{\partial w}{\partial z}\dfrac{dz}{dt} = 165t^{32}$.

Q5. $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\dfrac{dy}{dt} + \dfrac{\partial w}{\partial z}\dfrac{dz}{dt} = \dfrac{3 - (4/3)t^{-1/3} - 24t^{-7}}{3t - 2t^{2/3} + 4t^{-6}}$.

Q6. $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\dfrac{dy}{dt} + \dfrac{\partial w}{\partial z}\dfrac{dz}{dt} = -2t\cos\left(t^2\right)$.

Q7. $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\dfrac{dy}{dt} + \dfrac{\partial w}{\partial z}\dfrac{dz}{dt} = \dfrac{1 - 512t^5 - 2560t^5\ln t}{2t\sqrt{1 + \ln t - 512t^5\ln t}}$.

Q8. $\partial w/\partial x = (dw/d\rho)(\partial\rho/\partial x) = (x/\rho)dw/d\rho$, similarly $\partial w/\partial y = (y/\rho)dw/d\rho$ and $\partial w/\partial z = (z/\rho)dw/d\rho$ so $(\partial w/\partial x)^2 + (\partial w/\partial y)^2 + (\partial w/\partial z)^2 = (dw/d\rho)^2$.

Q9. $\dfrac{dw}{dt} = \dfrac{\partial w}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial w}{\partial y}\dfrac{dy}{dt} + \dfrac{\partial w}{\partial z}\dfrac{dz}{dt} = 1 + 2(\pi\cos\pi t)_{t=1} + 3(2t)_{t=1} = 1 - 2\pi + 6 = 7 - 2\pi$.

Q10. (a) $\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r} = [e^{x/y}(y + x)][\cos\theta] + [e^{x/y}(x - x^2/y)][\sin\theta] = re^{\cot\theta}[\sin 2\theta]$. Therefore,

$$\left.\frac{\partial z}{\partial r}\right|_{r=2,\theta=\pi/6} = 2e^{\cot(\pi/6)}sin(\pi/3) = \sqrt{3}e^{\sqrt{3}}$$

(b) $\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta} = [e^{x/y}(y+x)][-r\sin\theta] + [e^{x/y}(x - x^2/y)][r\cos\theta] = r^2e^{\cot\theta}[\cos 2\theta - \cot\theta]$.

$$\left.\frac{\partial z}{\partial \theta}\right|_{r=2,\theta=\pi/6} = 4e^{\cot(\pi/6)}[\cos(\pi/3) - \cot(\pi/6)] = 2(1 - 2\sqrt{3})e^{\sqrt{3}}.$$