National University of Technology, Islamabad

Assignment III (Calculus II), Spring 2019

Due Date: May 10, 2019

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Q.1 Show that the limit does not exist by considering the limits as $(x, y) \to (0, 0)$ along the coordinate axes.

(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x-y)}{(x^2+y^2)}$ (b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\cos(xy)}{(x^2+y^2)}$.

Q.2 Show that the value of $\dfrac{x^3y}{2x^6+y^2}$ approaches 0 as $(x, y) \to (0, 0)$ along any straight line $y = mx$, or along any parabola $y = kx^2$ for arbitrary $m, k \in \mathbb{R}$ . Also, show that $\dfrac{x^3y}{2x^6+y^2}$ does not exist by letting $(x, y) \to (0, 0)$ along the curve $y = x^3$.

Q.3 According to the ideal gas law, the pressure, temperature, and volume of a gas are related by $P = kT/V$, where $k$ is a constant of proportionality. Suppose that $V$ is measured in cubic inches $(in^3)$, $T$ is measured in Kelvins $(K)$, and that for a certain gas the constant of proportionality is $k = 10in.lb/K$.

(a) Find the instantaneous rate of change of pressure with respect to temperature if the temperature is $80K$ and the volume remains fixed at $50in^3$.

(b) Find the instantaneous rate of change of volume with respect to pressure if the volume is $50in^3$ and the temperature remains fixed at $80K$.

**In Q.** $4-7$ **use an appropriate form of the chain rule to find** $\dfrac{dw}{dt}$ **when:**

Q.4 $w = 5x^2y^3z^4, \quad x = t^2, \quad y = t^3, \quad z = t^5.$

Q.5 $w = \ln(3x^2 - 2y + 4z^3), \quad x = t^{1/2}, \quad y = t^{2/3}, \quad z = t^{-2}.$

Q.6 $w = 5\cos(xy) - \sin(xz), \quad x = 1/t, \quad y = t, \quad z = t^3.$

Q.7 $w = \sqrt{1 + x - 2xyz^4}, \quad x = \ln t, \quad y = t, \quad z = 4t.$

Q.8 Let $f$ be a differentiable function, and let $w = f(\rho)$, where $\rho = (x^2 + y^2 + z^2)^{1/2}$. Show that

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 = \left(\frac{dw}{d\rho}\right)^2.$$

Q.9 Suppose that $w = f(x, y, z)$ is differentiable at the point $(1, 0, 2)$ with $f_x(1, 0, 2) = 1$, $f_y(1, 0, 2) = 2$, and $f_z(1, 0, 2) = 3$. If $x = t$, $y = \sin(\pi t)$, and $z = t^2 + 1$, find $dw/dt$ when $t = 1$.

Q.10 Use the chain rule to find the values of $\left.\dfrac{\partial z}{\partial r}\right|_{r=2,\theta=\pi/6}$ and $\left.\dfrac{\partial z}{\partial \theta}\right|_{r=2,\theta=\pi/6}$ if $z = xye^{x/y}$ where $x = r\cos\theta$ and $y = r\sin\theta$.