Q.1 There are numerous applications in which the motion of an object must be controlled so that it moves towards a heat source. For example, in medical applications the operation of certain diagnostic equipment is designed to locate heat sources generated by tumors or infections and in military applications the trajectories of heat-seeking missiles (such as Stinger and Sidewinder) are controlled to seek and destroy enemy aircraft. In fact, these missiles use infrared sensors to measure gradients. In the following problem, you need to work out how gradients are used to solve such problems.

A heat-seeking particle is located at the point P(1,4) on a flat metal plate whose temperature at a point (x, y) is $T(x, y) = 5 - 4x^2 - y^2$. Find parametric equations for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

Q.2 Determine the dimensions of a rectangular box, open at the top, having a volume of $32\,ft^3$, and requiring the least amount of material (i.e., surface area) for its construction.

Q.3 Find three positive numbers whose sum is 27 and such that the sum of their squares is as small as possible.

Q.4 A manufacturer makes two models of an item, standard and deluxe. It costs Rs. 40 to manufacture the standard model and Rs 60 for the deluxe. A market research firm estimates that if the standard model is priced at x Rs. and the deluxe at y Rs., then the manufacturer will sell $500(y - x)$ of the standard items and $45000 + 500(x - 2y)$ of the deluxe each years. How should the items be priced to maximize the profit?

Q.5 Find the points on the surface $x^2 - yz = 5$ that are closest to the origin.

Q.6 Find all points on the portion of the plane $x + y + z = 5$ in the first octant at which $f(x, y, z) = xy^2z^2$ has a maximum value.

"There are two types of people who will tell you that you cannot make a difference in this world: those who are afraid to try and those who are afraid you will succeed." ~ Ray Goforth