National University of Technology, Islamabad
Assignment I (Calculus II), Spring 2019
Solution Key

Q1. If $|x|$ is the magnitude of the x-component, then $\cos 45^\circ = \frac{|x|}{|F|} \Rightarrow |x| = |F| \cos 45^\circ = (12)\left(\frac{\sqrt{2}}{2}\right) = 6\sqrt{2}$ lb

$\Rightarrow \mathbf{F}_x = -6\sqrt{2}\,\mathbf{i}$ (the negative sign is indicated by the diagram)

if $|y|$ is the magnitude of the y-component, then $\sin 45^\circ = \frac{|y|}{|F|} \Rightarrow |y| = |F| \sin 45^\circ = (12)\left(\frac{\sqrt{2}}{2}\right) = 6\sqrt{2}$ lb

$\Rightarrow \mathbf{F}_y = -6\sqrt{2}\,\mathbf{j}$ (the negative sign is indicated by the diagram)

Q2. (a) The tree is located at the tip of the vector $\overrightarrow{OP} = (5\cos 60^\circ)\mathbf{i} + (5\sin 60^\circ)\mathbf{j} = \frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j} \Rightarrow P = \left(\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

(b) The telephone pole is located at the point Q, which is the tip of the vector $\overrightarrow{OP} + \overrightarrow{PQ}$

$= \left(\frac{5}{2}\mathbf{i} + \frac{5\sqrt{3}}{2}\mathbf{j}\right) + (10\cos 315^\circ)\mathbf{i} + (10\sin 315^\circ)\mathbf{j} = \left(\frac{5}{2} + \frac{10\sqrt{2}}{2}\right)\mathbf{i} + \left(\frac{5\sqrt{3}}{2} - \frac{10\sqrt{2}}{2}\right)\mathbf{j}$

$\Rightarrow Q = \left(\frac{5+10\sqrt{2}}{2}, \frac{5\sqrt{3}-10\sqrt{2}}{2}\right)$

Q3. $|\mathbf{w}|\cos(33^\circ - 15^\circ) = 2.5$ lb, so $|\mathbf{w}| = \frac{2.5 \text{ lb}}{\cos 18^\circ}$. Then $\mathbf{w} = \frac{2.5 \text{ lb}}{\cos 18^\circ}\langle \cos 33^\circ, \sin 33^\circ \rangle \approx \langle 2.205, 1.432 \rangle$

Q4. $\mathbf{W} = |\mathbf{F}|\left|\overrightarrow{PQ}\right| \cos\theta = (200)(20)(\cos 30^\circ) = 2000\sqrt{3} = 3464.10 \text{ N} \cdot \text{m} = 3464.10 \text{ J}$

Q5. (a) $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\mathbf{i} + \mathbf{j} \Rightarrow \text{Area} = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \frac{1}{2}\sqrt{1+1} = \frac{\sqrt{2}}{2}$

(b) $\mathbf{u} = \pm \frac{\overrightarrow{PQ} \times \overrightarrow{PR}}{\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|} = \pm \frac{1}{\sqrt{2}}(-\mathbf{i} + \mathbf{j}) = \pm \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j})$

Q6. $\left|\overrightarrow{PQ} \times \mathbf{F}\right| = \left|\overrightarrow{PQ}\right| |\mathbf{F}| \sin(60^\circ) = \frac{2}{3} \cdot 30 \cdot \frac{\sqrt{3}}{2} \text{ ft} \cdot \text{lb} = 10\sqrt{3} \text{ ft} \cdot \text{lb}$

$\left|\overrightarrow{PQ} \times \mathbf{F}\right| = \left|\overrightarrow{PQ}\right| |\mathbf{F}| \sin(135^\circ) = \frac{2}{3} \cdot 30 \cdot \frac{\sqrt{2}}{2} \text{ ft} \cdot \text{lb} = 10\sqrt{2} \text{ ft} \cdot \text{lb}$