NUST

NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY

COURSE CODE:	MATH-322
COURSE NAME:	Group Theory
CREDIT HOURS:	(3,0)
CONTACT HOURS:	48
MODE OF TEACHING:	Lectures and Problem-Solving Activities
DEPARTMENT:	Mathematics
SCHOOL:	School of Natural Sciences (SNS)

COURSE DESCRIPTION:

Algebra, developed by the Muslims, started as generalized arithmetic but went on to deal with linear, quadratic, cubic and quartic equations and systems of such equations. Later, Galois and Abel developed group theory to prove that there is no canonical solution of higher order polynomial equations by means of radicals. Two further branches of development followed the solution of simultaneous linear equations (leading to linear algebra) and the more formal structure of groups and their extensions (leading to rings and fields). This course presents the basic concepts of group theory. The core contents discussed in this course will be:

- Sets and relations;
- Groups and Subgroups;
- Cyclic groups;
- Symmetric groups;
- Normal subgroups;
- Group homomorphism;
- Factor groups.

COURSE OBJECTIVES:

The main course objectives are to transmit core concepts of group theory from the foundations and to lead students to rigor in understanding mathematical structure of sets induced by binary operators in the form of group structures.

COURSE OUTCOMES:

On successful completion of this course, the students will be able to understand

1. Equivalence relations on sets, equivalence classes, partition of set into equivalence classes,
2. Binary operation, group, subgroup, order of a group, order of an element of a group, cyclic group,
3. Dihedral group, quaternion group, group of n^{th} roots of unity, group of residues, general linear group,
4. Permutation, group of permutations, orbits of a permutation cycles, transpositions, even and odd permutations,
5. Alternating group, decomposition of a permutation into disjoint cycles,
6. Coset, index of a subgroup in a group, Lagrange's theorem,

7. Normal subgroups, center subgroup, commutator subgroup,
8. Group homomorphism, kernel of a homomorphism, isomorphism,
9. Factor group, isomorphism theorems,
10. Product of subgroups.

TEXT AND MATERIAL:

Textbook(s)

1. J. H. Fraleigh, A First Course in Abstract Algebra, 7th ed., Addison-Wesley Publishing, 1998.

Reference Book(s):

2. J. A. Gallian, Contemporary Abstract Algebra, 7th ed., Brooks & Cole, Belmont, CA, 2009.
3. W. Keith Nicholson, Introduction to Abstract Algebra, 3rd ed., John Wiley & Sons, 2007.
4. I. N. Herstein, Abstract Algebra, 3rd ed., Prentice Hall, 1995.

References Books for Self-Study:

ASSESSMENT SYSTEM:

There will be 4 (reasonably brief but conceptual) Homework Assignments, 4 Quizzes, 2 OHTs, and 1 Final Exam. Date of submission of homework assignments and quizzes is reflected in the weekly schedule. Late submission will have a penalty (a deduction of 20% per day of late submission with a maximum deduction of 60% marks, however, a deduction of 100% for a delay of more than 5 days). To encourage reading and to discourage copying of homework assignments, all quizzes will have two parts:

i) **Reading Quiz:** It will be from reading assignments reflected in the course schedule. The results of this part will contribute towards the marks of the entire quiz.
ii) **Assignment Quiz:** It will be from the problem set in homework assignments. The result of this part will contribute towards 50% marks of the corresponding homework assignment.

Grading system as per the policy of NUST will be followed for the award of letter grades. Following weightages are assigned:

Assessments	No	Percentage
Assignments	4	10%
Quizzes	4	10%
OHT 1	1	15%
OHT 2	1	15%
Final Exam	1	50%

CONDUCT IN THE CLASS & IMPORTANT NOTES:

- A conducive and healthy classroom environment is paramount for learning. Discussion is highly encouraged during the lectures but the maintenance of the decorum must be ensured.
- Everyone is expected to be punctual. Consistent tardiness will be marked absence from the lecture after a fair warning.
- The attendance policy of NUST will be strictly adhered. **Make a clear note** that an absence from the class is strongly discouraged, and that up to 25% absentees tolerated by the policy are **ONLY** for tending to emergencies (hospitalization, etc.) and for inevitable personal engagements.
- Use of Mobile Phones/Smart Electronic Devices is not allowed during lectures. Such devices must be on silent mode.

INSTRUCTOR:

Name:	Dr. Abdul Wahab
Office:	Department of Mathematics, SNS Main Building, 1st floor, H12-Campus, NUST
Email:	abdul.wahab@sns.nust.edu.pk
Class Hours:	Consult timetable issued by the Department for lecture timings.
Office Hours:	Every Tuesday from 1600 – 1700Hrs (or by appointment).

INSTRUCTOR'S EXPERIENCE:

I was trained to be an Applied Mathematician. I did my PhD from École Polytechnique, Paris, and have been affiliated to University Denis Diderot (PARIS VII) as a Post-doc and Korea Advanced Institute of Science and Technology (KAIST), S. Korea as an Assistant Professor in the Department of Bio and Brain Engineering. I have research interests in direct and inverse scattering of waves in complex media, and mathematical imaging. I seek solutions to inverse problems related to biomedical imaging, non-destructive testing, exploration geophysics invisibility cloaking, and synergy of machine learning and inverse problems. I have co-authored a monograph on **Mathematical Methods in Elasticity Imaging**, which was published by *Princeton University Press, New Jersey, USA* in 2015.

I have vast experience of teaching a number of basic and advanced courses to both Math Majors and Engineering Majors including Linear Algebra, Ordinary Differential Equations, Partial Differential Equations, Topology, Numerical Analysis, Calculus and Analytic Geometry, Discrete Mathematics, and Mathematical Methods for Physics.

DETAILED COURSE OUTLINE AND WEEKLY SCHEDULE:

Week	Topics Covered	Reading Assignment	Assessment
1	Sets and relations, Partitions and equivalence relations, definitions, examples and related results.	Sect. 0 – Sect. 1 PS. 0-1	
2	Binary operations, groups, definitions and examples.	Sect. 2 – Sect 4 PS. 2-3	
3	Dihedral group, quaternion group, group of nth roots of unity, group of residues, general linear group.	Sect. 4 PS. 4	
4	Subgroups, order of a group, order of an element of a group, cyclic groups, cyclic subgroups, definitions, examples and related results, generating sets.	Sect. 5 – Sect 7 PS. 5-7	
5	Permutation, Group of permutations, examples, Cayley's theorem.	Sect. 8 – Sect 9 PS. 8	
6	OHT Week		**OHT – 1**
7	Orbits of a permutation. Cycles, transpositions, even and odd permutations.	Sect. 9 PS. 9	HWA – 1
8	Alternating group, decomposition of a permutation into disjoint cycles.	Sect. 9 PS. 9	Quiz – 1
9	Cosets, index of a subgroup in a group, definition examples related results, and the theorem of Lagrange.	Sect. 10 PS. 10	
10	Consequences of Lagrange's theorem, the index of a subgroup in a group.	Sect. 10 PS. 10	HWA – 2
11	Product of subgroups	Sect. 11 PS. 11	Quiz – 2
12	OHT Week		**OHT – 2**
13	Normal subgroups, definitions, examples and related results.	Sect. 13 – Sect 14 PS. 13	
14	Group homomorphism, Properties of a homomorphism, kernel of a homomorphism, definitions, examples and related results.	Sect. 14, PS. 14	HWA – 3
15	Isomorphism of groups, factor groups, factor groups from a homomorphism.	Sect. 14 PS. 14	Quiz – 3
16	Factor groups from normal groups, The fundamental homomorphism theorem.	Sect. 14 PS. 14	HWA – 4

17	Center and commutator subgroups.	Sect. 15 PS. 15	Quiz – 4
			Final Exam

Important Notes: *-Sections refer to the sections in the textbook (reference 1).*
-In the schedule, HW Assignments are mentioned in front of the week of their announcement.
-All HW Assignments will be due within one week after announcement.
-Solution Keys to HW Assignments will be posted one week after the deadline (i.e. two weeks after announcement).
-Solution Keys to Quizzes, OHT's and Final Exam will be posted next day after the conduct.
-PS = Problem Set / Exercise
-HWA=Homework Assignment.