NUST

NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY

COURSE CODE:	MATH-455
COURSE NAME:	Integral Equations
CREDIT HOURS:	(3,0)
CONTACT HOURS:	48
MODE OF TEACHING:	Lectures and Problem-Solving Activities
DEPARTMENT:	Mathematics
SCHOOL:	School of Natural Sciences (SNS)

COURSE DESCRIPTION:

This is a typical course to lead students through a traditional sequence of topics related to integral equations. This course emphasizes concepts and techniques for solving integral equations. Material is selected from the following topics:

- Classification of integral equations;
- Connection with differential equations;
- Integral equations of the convolution type;
- Method of successive approximations;
- Integral equations with singular kernels;
- Fredholm theory;
- Hilbert – Schmidt theory.

COURSE OBJECTIVES:

Integral equations have been of considerable significance in the history of mathematics. This course is mainly concerned with linear integral equations and a brief discussion of a simple type of non-linear integral equations, and the connection between differential and integral equations.

COURSE OUTCOMES:

On successful completion of this course, the students will be able to

1. Understand the theory of linear integral equations,
2. Apply different techniques to solve integral equations,
3. Understand the connection between differential and integral equations.

TEXT AND MATERIAL:

Textbook(s)

1. B. L. Moiseiwitsch, Integral Equations, Longman London and New York, 1977.

Reference Book(s):

2. H. Hochstadt, Integral Equations, John Wiley and Sons, 1973.

3. R. P. Kanwal, Linear Integral Equations: Theory and Technique, Academic Press, 1971.
4. Abdul-Majid Wazwaz, Linear and Nonlinear Integral Equations: Methods and Applications, Springer 2011.

References Books for Self-Study:

5. C. Corduneanu, Integral Equations and Applications, Cambridge University Press, 1991.
6. A. C. Pipkin, A Course on Integral Equations, Springer, New York, 1991.

ASSESSMENT SYSTEM:

There will be 6 (reasonably brief but conceptual) Homework Assignments, 6 Quizzes, 2 OHTs, and 1 Final Exam. Date of submission of homework assignments and quizzes is reflected in the weekly schedule. Late submission will have a penalty (a deduction of 20% per day of late submission with a maximum deduction of 60% marks, however, a deduction of 100% for a delay of more than 5 days). To encourage reading and to discourage copying of homework assignments, all quizzes will have two parts:

i) **Reading Quiz:** It will be from reading assignments reflected in the course schedule. The results of this part will contribute towards the marks of the entire quiz.
ii) **Assignment Quiz:** It will be from the problem set in homework assignments. The result of this part will contribute towards 50% marks of the corresponding homework assignment.

Grading system as per the policy of NUST will be followed for the award of letter grades. Following weightages are assigned:

Assessments	No	Percentage
Assignments	6	10%
Quizzes	6	10%
OHT 1	1	15%
OHT 2	1	15%
Final Exam	1	50%

CONDUCT IN THE CLASS & IMPORTANT NOTES:

- A conducive and healthy classroom environment is paramount for learning. Discussion is highly encouraged during the lectures but the maintenance of the decorum must be ensured.

- Everyone is expected to be punctual. Consistent tardiness will be marked absence from the lecture after a fair warning.
- The attendance policy of NUST will be strictly adhered. **Make a clear note** that an absence from the class is strongly discouraged, and that up to 25% absentees tolerated by the policy are **ONLY** for tending to emergencies (hospitalization, etc.) and for inevitable personal engagements.
- Use of Mobile Phones/Smart Electronic Devices is not allowed during lectures. Such devices must be on silent mode.

INSTRUCTOR:

Name: Dr. Abdul Wahab
Office: Department of Mathematics, SNS Main Building, 1st floor, H12-Campus, NUST
Email: abdul.wahab@sns.nust.edu.pk
Class Hours: Consult timetable issued by the Department for lecture timings.
Office Hours: Every Tuesday from 1400 – 1500Hrs (or by appointment).

INSTRUCTOR'S EXPERIENCE:

I was trained to be an Applied Mathematician. I did my PhD from École Polytechnique, Paris, and have been affiliated to University Denis Diderot (PARIS VII) as a Post-doc and Korea Advanced Institute of Science and Technology (KAIST), S. Korea as an Assistant Professor in the Department of Bio and Brain Engineering. I have research interests in direct and inverse scattering of waves in complex media, and mathematical imaging. I seek solutions to inverse problems related to biomedical imaging, non-destructive testing, exploration geophysics invisibility cloaking, and synergy of machine learning and inverse problems. I have co-authored a monograph on **Mathematical Methods in Elasticity Imaging**, which was published by *Princeton University Press, New Jersey, USA* in 2015.

I have vast experience of teaching a number of basic and advanced courses to both Math Majors and Engineering Majors including Linear Algebra, Ordinary Differential Equations, Partial Differential Equations, Topology, Numerical Analysis, Calculus and Analytic Geometry, Discrete Mathematics, and Mathematical Methods for Physics.

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DETAILED COURSE OUTLINE AND WEEKLY SCHEDULE:

Week	Topics Covered	Reading Assignment	Assessment
1	Classification of integral equations, Historical introduction, Linear integral equations, Special types of kernel, Symmetric kernels, Kernels producing convolution integrals, Separable kernels.	Sect. 1.1 – Sect. 1.3 PS. 1	
2	Square integrable functions and kernels, Singular integral equations, Non-linear equations.	Sect. 1.4 – Sect 1.6 PS. 1	HWA – 1
3	Linear differential equations, Green's function, Influence function.	Sect. 2.1 – Sect 2.3 PS. 2	Quiz – 1
4	Integral transforms, Fredholm equation of the second kind, Volterra equation of the second kind.	Sect. 3.1 – Sect 3.3 PS. 3	HWA – 2
5	Fredholm equations of the first kind, Stieltjes integral equations, Volterra equation of the first kind.	Sect. 3.4 – Sect 3.6 PS. 3	Quiz – 2
6	OHT Week		**OHT – 1**
7	Method of successive approximations: Neumann series, Iterates and the resolvent kernel	Sect. 4.1 – Sect. 4.2 PS. 4	
8	Generalization to higher dimensions, Green's functions in two and three dimensions.	Sect. 5.1 – Sect 5.2 PS. 5	HWA – 3
9	Dirichlet's problem, Poisson's formula for the unit disc, Poisson's formula for the half plane, Hilbert kernel, Hilbert transforms, Singular integral equations of Hilbert type.	Sect. 5.3 – Sect. 5.4 PS. 5	Quiz – 3
10	Resolvent equation, Uniqueness theorem, Characteristic values and functions.	Sect. 8.1 – Sect. 8.3 PS. 8	HWA – 4
11	Neumann series, Volterra integral equation of the second kind, Bacher's example, Fredholm equation in abstract Hilbert space.	Sect. 8.4– Sect 8.5 PS. 8	Quiz – 4
12	OHT Week		**OHT – 2**

13	Degenerate kernels, Approximation by degenerate kernels.	Sect. 9.1 – Sect 9.2 PS. 9	
14	Fredholm, theorems, Fredholm theorems for completely continuous, Operators, Fredholm formulae for continuous kernels.	Sect. 9.3 – Sect 9.4, PS. 9	HWA – 5
15	Hermitian kernels, Spectrum of a Hilbert-Schmidt kernel.	Sect. 10.1 – Sect. 10.2 PS. 10	Quiz – 5
16	Expansion theorems, Hilbert-Schmidt theorem, Hilbert's formula, Expansion theorem for iterated kernels, Solution of Fredholm equation of second kind.	Sect. 10.3 – Sect. 10.4 PS. 10	HWA – 6
17	Bounds on characteristic values, Positive kernels, Mercer's theorem, Variational principles, Rayleigh-Ritz variational method.	Sect. 10.5 – Sect. 10.8 PS. 10	Quiz – 6
			Final Exam

Important Notes: *-Sections refer to the sections in the textbook (reference 1).*
-In the schedule, HW Assignments are mentioned in front of the week of their announcement.
-All HW Assignments will be due within one week after announcement.
-Solution Keys to HW Assignments will be posted one week after the deadline (i.e. two weeks after announcement).
-Solution Keys to Quizzes, OHT's and Final Exam will be posted next day after the conduct.
-PS = Problem Set
-HWA=Homework Assignment.