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Electromagnetic time reversal and scattering by a small dielectric inclusion

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Abstract. An asymptotic expression of the scattered electromagnetic far-field due to a small spherical dielectric inclusion illuminated by an impulsive vector source nearby is proposed first. Then, a time-reversal technique enabling to locate the position of this inclusion is developed and its performances are illustrated from numerical simulations.

1. Introduction

In previous works, e.g., [4], asymptotic formulas of the electromagnetic fields scattered by a small inclusion (dielectric, magnetic, or perfectly conducting as well) or a finite collection of them have been developed in the frequency domain up to leading order vs. frequency and size of the inclusion. In acoustics and elasticity, similar asymptotic expansions have been rigorously established for both near- and far-fields scattered by similar inclusions in the frequency domain, again at leading order. From these expansions, one can investigate transient-wave cases by truncating (to summarize) the high frequencies and using inverse Fourier transforms, [1, 2]. This approach has been investigated and numerical simulations proposed in [9].

In this contribution, the electromagnetic case is investigated. A small 3-D bounded inclusion D with ideal dielectric permittivity ϵ and air magnetic permeability μ_0 (no magnetic behavior is set forth but the analysis extends to it) is considered, letting $D = \delta B + \mathbf{z}$, where B is a regular enough bounded domain in $\mathbb{R}^3$ which is representing the volume of the inclusion, $\mathbf{z}$ is the vector position of its center, and δ is the scale factor. This inclusion is located in the background (air) medium with air permittivity ϵ_0, letting $c = 1/\sqrt{\mu_0\epsilon_0}$ as the speed of light. It is illuminated by a point electric dipole at location $\bar{\mathbf{y}}$ assumed far away from $\mathbf{z}$. The question is threefold: Can one get a meaningful asymptotic expression of the scattered magnetic far-field in the time-harmonic regime? How does the corresponding formulation develop in the time-domain? Is time reversal as an imaging method a good mean to locate the inclusion?

The organization of the contribution is in parallel to the questioning. First, the asymptotic formula of the scattered magnetic far-field in frequency is introduced. Then, the one in time, by truncating high frequencies and applying the inverse Fourier transform, is proposed. Finally, time-reversal is developed and numerically illustrated as a method of localization of the inclusion.

2. Asymptotic formula of the scattered magnetic field in frequency domain

As already said, an ideal electric dipole with $\mathbf{e}$ vector direction is set at $\bar{\mathbf{y}}$ in air, and it is taken as the source of the illumination. One defines $\mathbf{J}_e = \mathbf{e}\delta(\mathbf{r}-\bar{\mathbf{y}})$ as the electric current density in space. At circular frequency ω (time-harmonic dependence as $+j\omega t$), the time-harmonic scattering problem in the presence of the inclusion reads as

$$\begin{cases} \nabla\times\left(\frac{1}{\epsilon_0}\nabla\times\mathbf{H}\right)-\omega^2\mu_0\mathbf{H}=\nabla\times\frac{1}{\epsilon_0}\mathbf{J}_e \quad \text{in} \quad \mathbb{R}^3\setminus D \\ \nabla\times\left(\frac{1}{\epsilon}\nabla\times\mathbf{H}\right)-\omega^2\mu_0\mathbf{H}=0 \quad \text{in} \quad \bar{D} \\ \frac{1}{\epsilon_0}\left(\nabla\times\mathbf{H}\right)^+\times\nu-\frac{1}{\epsilon}\left(\nabla\times\mathbf{H}\right)^-\times\nu=0 \quad \text{on} \quad \partial D \\ \mu_0\mathbf{H}^+\cdot\nu-\mu_0\mathbf{H}^-\cdot\nu=0 \quad \text{on} \quad \partial D \end{cases}$$

Here ν is the unit normal on ∂D, exponents $^+$ and $^-$ referring to limit values from the outside and the inside as usual. $\underline{\mathbf{G}}^{me}$ is the magnetic-electric dyadic Green function in free space, satisfying the dyadic Helmholtz equation

$$\nabla\times\nabla\times\underline{\mathbf{G}}^{me}-\frac{\omega^2}{c^2}\underline{\mathbf{G}}^{me}=\nabla\times\underline{\mathbf{I}}\delta\left(\mathbf{r}\right). \tag{1}$$

with proper behavior to be accounted for at infinity. $\mathbf{H}_0$ is the incident magnetic field (the field due to the dipole in the absence of the inclusion) such as

$$\nabla\times\left(\frac{1}{\epsilon_0}\nabla\times\mathbf{H}_0\right)-\omega^2\mu_0\mathbf{H}_0=\nabla\times\tfrac{1}{\epsilon_0}\mathbf{J}_e \quad \text{in} \quad \mathbb{R}^3, \quad \text{and} \quad \nabla\cdot\mathbf{H}_0=0 \quad \text{in} \quad \mathbb{R}^3$$

or equivalently $\mathbf{H}_0(\mathbf{r},\bar{\mathbf{y}},\omega)=-\underline{\mathbf{G}}^{me}(\mathbf{r},\bar{\mathbf{y}},\omega)\cdot\mathbf{e}$, where $\underline{\mathbf{G}}^{me}(\mathbf{r},\bar{\mathbf{y}})=\nabla\times\underline{\mathbf{I}}\dfrac{e^{-i\frac{\omega}{c}|\mathbf{r}-\bar{\mathbf{y}}|}}{4\pi|\mathbf{r}-\bar{\mathbf{y}}|}$, using the notation $(\mathbf{r},\bar{\mathbf{y}},\omega)$ to mark the dependence upon the source dipole and frequency.

Let henceforth assume that the source point $\bar{\mathbf{y}}$ lies far enough from the inclusion, and that this is true as well for the observation point $\mathbf{x}$. One derives an asymptotic representation of the magnetic field in the frequency domain when $(\omega/c)\delta << 1$ [5]:

$$\left(\mathbf{H}-\mathbf{H}_0\right)\left(\mathbf{x},\bar{\mathbf{y}},\omega\right)\approx\delta^3\underline{\mathbf{G}}^{me}\left(\mathbf{x},\mathbf{z},\omega\right)\cdot\underline{\mathbf{M}}\left(\epsilon,B\right)\nabla\times\mathbf{H}_0\left(\mathbf{z},\bar{\mathbf{y}},\omega\right), \tag{2}$$

wherein the (frequency-independent) polarization tensor $\underline{\mathbf{M}}\left(\epsilon,B\right)$ in the case of a small dielectric spherical inclusion reduces to [3] $\underline{\mathbf{M}}\left(\epsilon,B\right)=3(\epsilon-\epsilon_0)(\epsilon+2\epsilon_0)^{-1}|B|\underline{\mathbf{I}}$, with immediate extensions to the general triaxial ellipsoid and oblate and prolate spheroids.

The asymptotic formula is given at leading order in terms of frequency and size of the inclusion. The latter should be small compared to the wavelength of the wave impinging upon it. Then, as known, it behaves as a radiative electric dipole.

3. Asymptotic formula of the scattered magnetic field in time domain

Now, one considers the time-domain counterpart of the previous results. The magnetic-electric dyadic Green function $\underline{\mathbf{G}}^{me}$ is solution of the dyadic Helmholtz equation

$$\nabla\times\nabla\times\underline{\mathbf{G}}^{me}+\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\underline{\mathbf{G}}^{me}=\nabla\times\underline{\mathbf{I}}\delta\left(\mathbf{r}\right)\delta\left(t\right) \tag{3}$$

with again proper behavior at infinity. One easily shows that

$$\underline{\mathbf{G}}^{me}\left(\mathbf{r},t\right)=\nabla\times\underline{\mathbf{I}}\frac{\delta\left(t-|\mathbf{r}|/c\right)}{4\pi|\mathbf{r}|}=\left[\frac{\delta\left(t-|\mathbf{r}|/c\right)}{|\mathbf{r}|}+\frac{\delta'\left(t-|\mathbf{r}|/c\right)}{c}\right]\frac{\hat{\mathbf{r}}\times\underline{\mathbf{I}}}{4\pi|\mathbf{r}|} \tag{4}$$

where $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$, $\delta'(\alpha) = d\delta(\alpha)/d(\alpha)$. As for the incident magnetic field radiated by an electric dipole source at $\bar{\mathbf{y}}$, it is given in the time domain by [8]:

$$\mathbf{H}_0(\mathbf{r}, \bar{\mathbf{y}}, t) = -\underline{\mathbf{G}}^{me}(\mathbf{r}, \bar{\mathbf{y}}, t) \cdot \mathbf{e}. \tag{5}$$

To derive the time-domain asymptotic formula, one needs to truncate the high frequencies. Once those higher than ω_c are taken off, the Dirac impulsion exciting the electric dipole becomes

$$\psi(t) = \int_{|\omega| \leq \omega_c} e^{j\omega t} \frac{d\omega}{2\pi} = \frac{\sin \omega_c t}{\pi t}. \tag{6}$$

The operator P_{ω_c} can be defined as a tempered distribution by $P_{\omega_c}[\psi](t) = \int_{|\omega| \leq \omega_c} \hat{\psi}(\omega) e^{j\omega t} \dfrac{d\omega}{2\pi}$. Applying the inverse Fourier transform of (2) after truncating the high frequencies, a truncated equation satisfied by the incident field follows as

$$\begin{cases} \nabla \times \left(\dfrac{1}{\epsilon_0} \nabla \times P_{\omega_c}[\mathbf{H}_0] \right) + \mu_0 \dfrac{\partial^2}{\partial t^2} P_{\omega_c}[\mathbf{H}_0] = \nabla \times \delta(\mathbf{r} - \bar{\mathbf{y}}) \psi(t) \mathbf{e} & \text{in} \quad \mathbb{R}^3 \times \mathbb{R} \\ \nabla \cdot P_{\omega_c}[\mathbf{H}_0] = 0 \quad \text{in} \quad \mathbb{R}^3 \times \mathbb{R} \end{cases}$$

If the point of observation is away from the inclusion, the asymptotic formula of the scattered magnetic field, when $\omega << c/\delta$, can be written as :

$$P_{\omega_c}[\mathbf{H} - \mathbf{H}_0](\mathbf{x}, \bar{\mathbf{y}}, t) \approx \delta^3 \int_{\mathbb{R}} P_{\omega_c}[\underline{\mathbf{G}}^{me}](\mathbf{x}, \mathbf{z}, t - \tau) \cdot \underline{\mathbf{M}}(\epsilon, B) \nabla \times P_{\omega_c}[\mathbf{H}_0](\mathbf{z}, \bar{\mathbf{y}}, \tau)\, d\tau, \tag{7}$$

In the above, the truncated dyadic Green function $P_{\omega_c}[\underline{\mathbf{G}}^{me}]$ is

$$P_{\omega_c}[\underline{\mathbf{G}}^{me}](\mathbf{r}, t) = \left[\frac{\psi(t - |\mathbf{r}|/c)}{|\mathbf{r}|} + \frac{\psi'(t - |\mathbf{r}|/c)}{c} \right] \frac{\hat{\mathbf{r}} \times \underline{\mathbf{I}}}{4\pi|\mathbf{r}|}. \tag{8}$$

From (7), scattering by the inclusion is equivalent to radiation of an electric dipole with specific time-dependence.

The *sinc* signal is not causal. To avoid this non-causality problem, one could instead use as source pulse on the electric dipole (this will impact the incident field and thus the scattered field) a time-shifted *sinc* centred at large enough t_s so as to be close to zero at the time origin (computationally speaking, several orders of magnitude smaller than the peak value). One could also introduce a Gaussian pulse and shift it. A possibly better solution would be to use as source pulse the derivative of the Blackman-Harris one [10], to maintain causality while ensuring a reasonably wide band, truncation then being effected at ω_c large vs. the usable bandwidth.

4. Time reversal

The idea of time-reversal is to record the transient tangential components of the wavefield on a closed surface surrounding the inclusion, and to retransmit them onto the same background in time-reversed chronology. Then the wave should refocus at the location of the inclusion[7, 6].

So, one assumes that the tangential components of the fields are collected on a large sphere S with normal ν surrounding the inclusion D for a sufficiently large window of time t_0. Time-reversal is described by the transform $t \mapsto t_0 - t$. Then, both tangential components on S are time-reversed and emitted from S. A time-reversed field $\mathbf{H}_{\text{tr}}$ propagates into the interior volume. Using the asymptotic formula (7), one can prove that it is approximated by [2]

$$\begin{aligned} \mathbf{H}_{\text{tr}}(\mathbf{x}, \bar{\mathbf{y}}, t) \approx & \int_{\mathbb{R}} dt' \int_S \left[\underline{\mathbf{G}}^{ee}(\mathbf{x}, \mathbf{x}', t - t') \cdot \left(\nu \times \nabla \times P_{\omega_c}[\mathbf{H} - \mathbf{H}_0](\mathbf{x}', \bar{\mathbf{y}}, t_0 - t') \right) \right. \\ & \left. + \nabla \times \underline{\mathbf{G}}^{ee}(\mathbf{x}, \mathbf{x}', t - t') \cdot \left(\nu \times P_{\omega_c}[\mathbf{H} - \mathbf{H}_0](\mathbf{x}', \bar{\mathbf{y}}, t_0 - t') \right) \right] d\sigma(\mathbf{x}'). \end{aligned} \tag{9}$$

the dependence upon $\mathbf{z}$ in the reversed field being implied, where $\underline{\mathbf{G}}^{ee}$ is defined in [11].

One gets

$$\mathbf{H}_{\text{tr}}(\mathbf{x},\bar{\mathbf{y}},t) \approx -\delta^3 \int_{\mathbb{R}} d\tau \int_{\mathbb{R}} dt' \int_S \left[\underline{\mathbf{G}}^{ee}(\mathbf{x},\mathbf{x}',t-t') \cdot \left(\nu \times \nabla \times P_{\omega_c}[\underline{\mathbf{G}}^{me}](\mathbf{x}',\mathbf{z},t_0-\tau-t')\right) + \nabla \times \underline{\mathbf{G}}^{ee}(\mathbf{x},\mathbf{x}',t-s) \cdot \left(\nu \times P_{\omega_c}[\underline{\mathbf{G}}^{me}](\mathbf{x}',\mathbf{z},t_0-\tau-t')\right)\right] \cdot \mathbf{p}(\mathbf{z},\bar{\mathbf{y}},\tau) d\sigma(\mathbf{x}').$$

Since

$$\begin{aligned}&\int_{\mathbb{R}} dt' \int_S \left[\underline{\mathbf{G}}^{ee}(\mathbf{x},\mathbf{x}',t-t') \cdot \left(\nu \times \nabla \times P_{\omega_c}\left[\underline{\mathbf{G}}^{me}(\mathbf{x}',\mathbf{z},t_0-\tau-t')\right)\right] + \right.\\ &\left.\nabla \times \underline{\mathbf{G}}^{ee}(\mathbf{x},\mathbf{x}',t-t') \cdot \left(\nu \times P_{\omega_c}[\underline{\mathbf{G}}^{me}](\mathbf{x}',\mathbf{z},t_0-\tau-t')\right)\right] d\sigma(\mathbf{x}')\\ &= \mu_0 \frac{\partial}{\partial t}\left[P_{\omega_c}\left[\underline{\mathbf{G}}^{me}(\mathbf{x},\mathbf{z},t_0-\tau-t)\right] - P_{\omega_c}\left[\underline{\mathbf{G}}^{me}(\mathbf{x},\mathbf{z},t-t_0+\tau)\right]\right], \end{aligned} \tag{10}$$

one arrives at

$$\begin{aligned}\mathbf{H}_{\text{tr}}(\mathbf{x},\bar{\mathbf{y}},t) \approx -\delta^3 \int_{\mathbb{R}} \quad & \mu_0 \frac{\partial}{\partial t}\left[P_{\omega_c}\left[\underline{\mathbf{G}}^{me}(\mathbf{x},\mathbf{z},t_0-\tau-t)\right] - P_{\omega_c}\left[\underline{\mathbf{G}}^{me}(\mathbf{x},\mathbf{z},t-t_0+\tau)\right]\right]\\ & \cdot \mathbf{p}(\mathbf{z},\bar{\mathbf{y}},\tau)\, d\tau,\end{aligned} \tag{11}$$

where $\mathbf{p}(\mathbf{z},\bar{\mathbf{y}},\tau) = \underline{\mathbf{M}}(\epsilon,B) \cdot \nabla \times P_{\omega_c}[\mathbf{H}_0](\mathbf{z},\bar{\mathbf{y}},\tau)$.

The interpretation is that there is superposition of ingoing and outgoing waves, centered at location $\mathbf{z}$ of the inclusion. To show it more clearly, one assumes that $\mathbf{p}(\mathbf{z},\bar{\mathbf{y}},\tau)$ is concentrated at $\tau = T = |\mathbf{z}-\bar{\mathbf{y}}|/c$, which is reasonable since $\mathbf{p}(\mathbf{z},\bar{\mathbf{y}},\tau)$ peaks at $\tau = T$. Then, (11) becomes

$$\mathbf{H}_{\text{tr}}(\mathbf{x},\bar{\mathbf{y}},t) \approx -\delta^3 \mu_0 \frac{\partial}{\partial t}\left[P_{\omega_c}\left[\underline{\mathbf{G}}^{me}(\mathbf{x},\mathbf{z},t_0-T-t)\right] - P_{\omega_c}\left[\underline{\mathbf{G}}^{me}(\mathbf{x},\mathbf{z},t-t_0+T)\right]\right] \cdot \mathbf{p}(\mathbf{z},\bar{\mathbf{y}},T), \tag{12}$$

indeed exhibiting ingoing and outgoing spherical waves.

Taking the Fourier transform of (10) and (11), letting $*$ denote the conjugate, one obtains:

$$\begin{aligned}\mathbf{H}_{\text{tr}}(\mathbf{x},\bar{\mathbf{y}},\omega) &\approx \int_S \left[\underline{\mathbf{G}}^{ee}(\mathbf{x},\mathbf{x}',\omega) \cdot \left(\nu \times \nabla \times [\mathbf{H}-\mathbf{H}_0]^*(\mathbf{x}',\mathbf{z},\omega)\right)\right.\\ &\left. + \nabla \times \underline{\mathbf{G}}^{ee}(\mathbf{x},\mathbf{x}',\omega) \cdot \left(\nu \times [\mathbf{H}-\mathbf{H}_0]^*(\mathbf{x}',\mathbf{z},\omega)\right)\right] d\sigma(\mathbf{x}')\\ &= -2\omega\mu_0\delta^3 \Im m \underline{\mathbf{G}}^{me}(\mathbf{x},\mathbf{z},\omega) \cdot \underline{\mathbf{M}}(\epsilon,B) \nabla \times \mathbf{H}_0(\mathbf{z},\mathbf{y},\omega).\end{aligned} \tag{13}$$

The time-reversed field is proportional to the imaginary part of the Green dyad (manifesting the Rayleigh resolution, not withstanding polarization-dependent phenomena). Refer to classical results on the generalized Porter-Bojarski integral equation, e.g., among many, [12].

5. Numerical simulations

A dielectric inclusion of diameter $\lambda/25$ where $\lambda = 0.75$m and relative permittivity 3 is placed at the origin of space. It is illuminated by a point dipole at $(6\lambda = 4.5 \text{ m}, 0, 0)$ position (i.e., $T = 15$ns), with unit direction $\mathbf{e} = [0,1,0]$. All frequencies above 400 MHz are truncated. Figure 1 shows the excitation signal *sinc* shifted by $t_s = 20 * (2\pi/\omega_c) = 50$ns and the z-component of the scattered magnetic field computed by the asymptotic formula in the time domain as function of time (at the location of the source see Fig. 2). As expected, the scattered field is concentrated around the propagation time of the wave, at $t = 2T + t_s = 80$ns.

Time reversal is applied to locate the inclusion at instant $t = t_0 - T$ in the plane (x,y), where $t_0 = 140$ns (one is safely close to a 0 magnitude of the recorded field). Figure 3 indeed depicts an anti-symmetric focal spot at the position of the inclusion.

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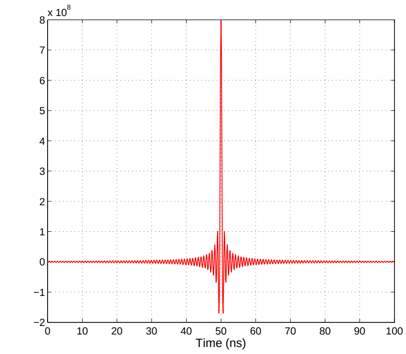

Figure 1. sinc excitation shifted by $t_s = 50$ns.

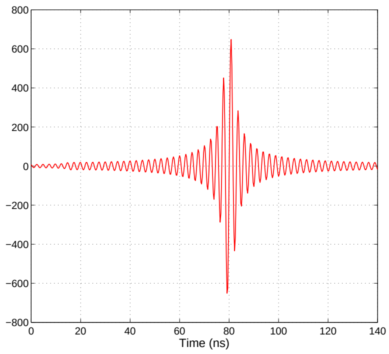

Figure 2. z-component of the scattered magnetic field.

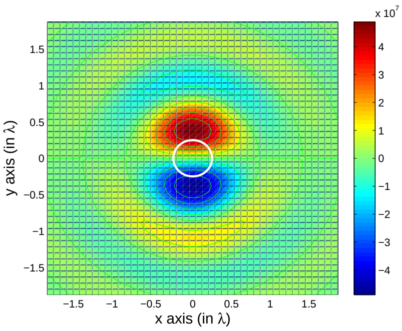

Figure 3. Time reversal of the z-component of the magnetic field in plane (x, y) at time $t = 125$ns.

Now, one excites the dipole by a causal signal as the first derivative of the Blackman-Harris pulse with central frequency $f_c = 100$ MHz where $\lambda_c = 3$m (to respect the bandwidth limitation), the duration of the source being $T_c = 1.55/f_c = 15.5$ns (see Fig. 4). Figure 5 shows that the z-component of the scattered field at the source location as a function of time is concentrated at instant $t = 37.75$ns with duration T_c .

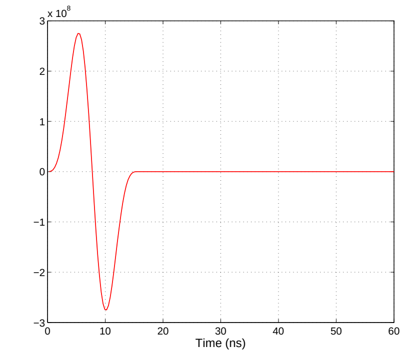

Figure 4. Derivative of Blackman-Harris signal excitation.

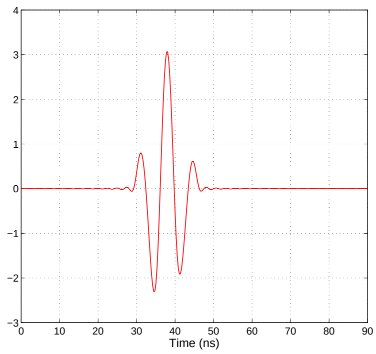

Figure 5. The z-component of the scattered magnetic field.

In figure 6, we plot the z-component of the time-reversed magnetic field at instant 67.25ns in the plane (x, y), letting $t_0 = 90$ns.

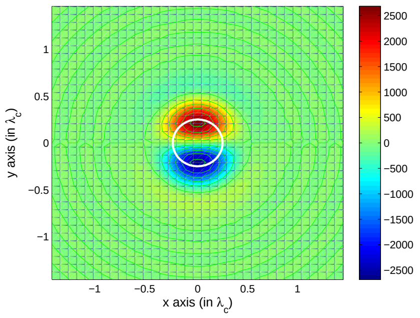

Figure 6. Time reversal of the z-component of the magnetic field in plane (x, y) at time $t = 67.25$ns.

6. Conclusion

Asymptotic formulas of the magnetic field (per duality, electric, if need be) scattered by a small dielectric inclusion in the time domain have been validated. Time-reversal enables to refocus to some extent onto the position of the inclusion. The next step is to study the effect of dispersion and attenuation of the background medium involved on the performance of this approach.

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