National University of Technology, Islamabad
Midterm (Calculus II), Spring 2019
BET (Civil), Instructor: Dr. Abdul Wahab
Date: May 10, 2019, Time: 60Min., Marks: 30.

Instructions: The use of electronic gadgets and any kind of notes or helping material is **strictly prohibited**. Do not write anything on the question paper other than your name or registration number. Return the question paper along with the answer sheet.

Q.1 Let the storage dome of the raw-material (mainly lime stone) at the quarry site of Fouji Cement Company be modeled by the function $f(x,y) = \sqrt{4-x^2-y^2}$. Calculate the domain of f that represents the region in Cartesian coordinates on which the storage dome is constructed. **Explain** the shape of the contour of the dome on the ground (i.e., the contour curve of f when $c = 0$)?
[SLO 1, PLO 1, Marks 3]

Q.2 A mason wants to decorate a small room with two different styles of marbles. The total cost of the decoration is modeled as $C = C(x,y) = x^2y - y^2$ where x, y are costs of individual marbles used in the process. At the same time, the prices of both the marbles fluctuate with time as $x = x(t) = e^t - \sinh t$ and $y = y(t) = \cosh t$. **Calculate** the total rate of change of the cost of decoration with time. Express your answer as a function of time.
[SLO 3, PLO 1, Marks 7]

Q.3 A flat table, modeled with a plane having equation $2x + 2y - z = 3$ is cut by a blade having equation $3x + 6z = 1$. **Find** the equation of the line of intersection between plane and the blade.
[SLO2, PLO 1, Marks 7]

Q.4 **Show** that $\lim\limits_{(x,y)\to(0,0)} \left(\dfrac{x^2-y^2}{x^2+y^2}\right)^2$ does not exist.
[SLO 1, PLO 1, Marks 5]

Q.5 **Calculate** all the second partial derivatives of $z := x^2y^2e^{2xy}$. Also verify the equality of the mixed derivatives.
[SLO 3, PLO 1, Marks 5]

Q.6 **Calculate** the directional derivative of $f(x,y) := e^x \cos(\pi y)$ in the direction of

$$\vec{v} = -\frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{j}$$

at point $(0,-1)$.
[SLO 1, PLO 1, Marks 3]

"Whether you think you can or think you can't, you're right" Henry Ford