SCHOOL OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS
OHT-1 (MATH-322 GROUP THEORY), SPRING 2020
TIME: 60 MINUTES, MARKS: 30

Instructions: Use of electronic gadgets, calculators or any course related material is not allowed. Do not write any thing on the question paper except your name and registration number.

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1. (a) Let $*$ be a binary operation over $\mathbb{Q}^+$ defined by

$$x * y = \frac{xy}{4}, \quad \forall x, y \in \mathbb{Q}^+.$$

Determine whether $(\mathbb{Q}^+, *)$ is a group or not? **(5 Points)**

(b) Let $(G, *)$ be a group. Prove that in group G there exist a unique idempotent element. **(5 Points)**

2. (a) Let $(G, *)$ be a group and $H, K \subset G$ be subgroups of G. Prove that $H \cap K \subset G$ is a subgroup. **(5 Points)**

(b) Let $(G, *)$ be a group. Prove that

$$(a * b)^{-1} = b^{-1} * a^{-1}, \quad \forall a, b \in G.$$

(5 Points)

3. (a) Let $(G, *)$, (H, o) and $(M, \bullet)$ be three groups. Let $f : G \to H$ and $g : H \to M$ be homomorphism. Prove that the composition $gf : G \to M$ is homomorphism. **(5 Points)**

(b) Let $(G, *)$ be a group and $H \subset G$ be subgroup of G. Let us define a relation on G using H as follows.

$$x, y \in G, \quad x \sim y \iff x^{-1} * y \in H. \qquad (*)$$

Prove that (*) gives an equivalence relation on G. **(5 Points)**

"Don't let what you *cannot do* interfere with what you *can do*" — John Wooden.