School of Natural Sciences

Department of Mathematics

OHT-1 (MATH-455 Integral Equations), Spring 2020

Time: 60 Minutes, Marks: 50

Name: ______________________ Reg. No.: __________ Sheet No.: __________

Instructions: Use of electronic gadgets, calculators or any course related material is not allowed. Solve Q.1 Part (a) on the question paper that has to be returned along with answer sheet.

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1. (a) Classify the integral equations as linear, non-linear, homogeneous, non-homogeneous, singular, non-singular, first kind, second kind, Volterra and Fredholm.
(2.5+2.5 Points)

(i) $1 + \frac{\varphi(x)}{\cos x} - \lambda \int_0^{\pi/3} \frac{\sin^2(x-t)\varphi(t)}{t^2} dt = 0.$

Ans.

(ii) $2\psi(x) + 3\int_0^7 M(x,t)\psi(t)dt = 0$, where $M(x,t) := \begin{cases} x^2 - t^2, & 0 \le t \le x, \\ t^2 + x^2, & x \le t \le 7. \end{cases}$

Ans.

(b) Show that $y(x) = \dfrac{1}{(1+x^2)^{3/2}}$ is a solution to the integral equation

$$y(x) = \frac{1}{(1+x^2)} - \int_0^x \frac{t}{(1+x^2)} y(t)dt. \qquad \textbf{(5 Points)} \qquad (1)$$

2. Find the spectrum of the integral equation

$$g(x) = \lambda \int_{-\pi}^{\pi} x \sin t \, g(t)dt, \qquad (2)$$

and eigen solutions. Discuss qualities of the spectrum (any two)?
(6+2+2 Points)

3. Solve and identify the resolvent kernel of the integral equation

$$g(x) = x + \lambda \int_{-\pi}^{\pi} x \sin t \, g(t)dt. \qquad \textbf{(7+3 Points)} \qquad (3)$$

4. Convert the integral equation

$$u(\xi) = \lambda \int_0^1 \kappa(\xi, t) u(t) dt, \tag{4}$$

with

$$\kappa(\xi, t) = \begin{cases} \xi(1-t), & \xi \leq t \leq 1, \\ t(1-\xi), & 0 \leq t \leq \xi, \end{cases} \tag{5}$$

to a boundary value problem with suitable boundary conditions. **(8+2 Points)**

5. Using the potential function $\varphi(x) := \dfrac{d^2v}{dx^2}$, form an integral equation corresponding to the initial value problem

$$\frac{d^2v}{dx^2} + x\frac{dv}{dx} + v = 0, \tag{6}$$

$$v(0) = 1, \tag{7}$$

$$\frac{dv}{dx}(0) = 0. \tag{8}$$

(10 Points)

"Don't let what you *cannot do* interfere with what you *can do*" — John Wooden.