



NATIONAL UNIVERSITY OF TECHNOLOGY, ISLAMABAD

QUIZ III (CALCULUS II), SPRING 2019

DATED: MAY 13, 2019

Q.1 Use the chain rule to find the values of $\frac{\partial z}{\partial r}\bigg|_{r=2, \theta=\pi/6}$ and $\frac{\partial z}{\partial \theta}\bigg|_{r=2, \theta=\pi/6}$ if $z = xye^{x/y}$ where $x = r \cos \theta$ and $y = r \sin \theta$.

Sol.

(a)

$$\begin{aligned}\frac{\partial z}{\partial r} &= \frac{\partial z}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial r} \\ &= [e^{x/y}(y+x)][\cos \theta] + [e^{x/y}(x - x^2/y)][\sin \theta] \\ &= re^{\cot \theta} [\sin 2\theta].\end{aligned}$$

Therefore,

$$\frac{\partial z}{\partial r}\bigg|_{r=2, \theta=\pi/6} = 2e^{\cot(\pi/6)} \sin(\pi/3) = \sqrt{3}e^{\sqrt{3}}$$

(b)

$$\begin{aligned}\frac{\partial z}{\partial \theta} &= \frac{\partial z}{\partial x} \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial \theta} \\ &= [e^{x/y}(y+x)][-r \sin \theta] + [e^{x/y}(x - x^2/y)][r \cos \theta] \\ &= r^2 e^{\cot \theta} [\cos 2\theta - \cot \theta].\end{aligned}$$

Therefore,

$$\frac{\partial z}{\partial \theta}\bigg|_{r=2, \theta=\pi/6} = 4e^{\cot(\pi/6)} [\cos(\pi/3) - \cot(\pi/6)] = 2(1 - 2\sqrt{3})e^{\sqrt{3}}.$$

“What seems to us as bitter trials are often blessings in disguise.” ~Oscar Wilde