NATIONAL UNIVERSITY OF TECHNOLOGY, ISLAMABAD
QUIZ IV (CALCULUS II), SPRING 2019
SOLUTION KEY

Q.1 Find three positive numbers whose sum is 48 and such that their product is as large as possible.

Sol. Let x, y, and z be the required numbers. We maximize $P = xyz$ subject to $x + y + z = 48$, $x > 0$, $y > 0$, and $z > 0$. Eliminating z by substituting $z = 48 - x - y$ we get

$$P(x, y) = xy(48 - x - y) = 48xy - x^2y - xy^2.$$

For second derivative test, we derive

$$P_x = 48y - 2xy - y^2, \quad P_y = 48x - x^2 - 2xy, \quad P_{xx} = -2y, \quad P_{yy} = -2x, \quad P_{xy} = 48 - 2x - 2y.$$

For critical points, set $P_x = 0 = P_y$ to get $y(48 - 2x - y) = 0$ and $x(48 - x - 2y) = 0$. Since $x > 0$ and $y > 0$, we have

$$2x + y = 48 \quad \text{and} \quad x + 2y = 48.$$

Solving these equations, we get the critical values $(x, y) = (16, 16)$. Since

$$H\Big|_{(16,16)} = \Big[P_{xx}P_{yy} - P_{xy}^2\Big]\Big|_{(16,16)} = \Big[4xy - (48 - 2x - 2y)^2\Big]\Big|_{(16,16)} = 3(16)^2 = 768 > 0,$$

and

$$P_{xx}(16, 16) = -2(16) = -32 < 0,$$

we have relative maximum at $x = 16$, $y = 16$, and $z = 48 - x - y = 16$. Therefore, for the numbers $x = y = z = 16$, the product is maximum while the sum is 48.

“Successful people do what unsuccessful people are not willing to do. Don’t wish it were easier, wish you were better” ∼ Jim Rohn