National University of Technology, Islamabad
Quiz IV (Calculus II), Spring 2019
Solution Key

Q.1 Find a vector in 3-dimensional space whose length is 5 and whose components have the largest possible sum.

Sol. Let $\mathbf{v} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ be the required vector such that $|\mathbf{v}| = 5$, i.e., $x^2 + y^2 + z^2 = 25$ and $f(x, y, z) = x + y + z$ is largest (f is the objective function here). The problem at hand is as follows. Maximize the function $f(x, y, z) = x + y + z$ subject to the constrain $g(x, y, z) = x^2 + y^2 + z^2 - 25$. Using the method of Lagrange multipliers, we set $\nabla f = \lambda \nabla g$, i.e.,

$$\mathbf{i} + \mathbf{j} + \mathbf{k} = \lambda(2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}) \iff 1 = 2x\lambda, \quad 1 = 2y\lambda, \quad 1 = 2z\lambda.$$

This implies

$$\lambda = \frac{1}{2x} = \frac{1}{2y} = \frac{1}{2z} \iff y = x \quad \text{and} \quad z = x.$$

With this information at hand, we have from the constraint equation

$$25 = x^2 + y^2 + z^2 = x^2 + x^2 + x^2 = 3x^2 \iff x = \pm\sqrt{25/3} = \pm 5/\sqrt{3} = y = z.$$

Since

$$f\left(\frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}, \frac{5}{\sqrt{3}}\right) = \frac{5}{\sqrt{3}} \quad \text{and} \quad f\left(-\frac{5}{\sqrt{3}}, -\frac{5}{\sqrt{3}}, -\frac{5}{\sqrt{3}}\right) = -\frac{5}{\sqrt{3}},$$

therefore, the required vector is $\mathbf{v} = \frac{5}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$.

"Doubt kills more dreams than failure ever will." —Suzy Kassem