Q.1 Integrate the function $f(x, y) = x^2 + y^2$ over the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$.

Ans. On can easily find out that the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$ is bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$. Therefore, over this region of integration, we have

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\begin{aligned}
\iint_R (x^2 + y^2) \, dy \, dx &= \int_0^1 \int_0^{1-x} (x^2 + y^2) \, dy \, dx \\
&= \int_0^1 \left[x^2 y + \frac{y^3}{3} \right]_0^{1-x} ds \\
&= \int_0^1 \left[x^2(1 - x) + \frac{(1 - x)^3}{3} \right] dx \\
&= \int_0^1 \left[x^2 - x^3 + \frac{(1 - x)^3}{3} \right] dx \\
&= \left[\frac{x^3}{3} - \frac{x^4}{4} - \frac{(1 - x)^4}{12} \right]_0^1 \\
&= \left(\frac{1}{3} - \frac{1}{4} - 0 \right) - \left(0 - 0 - \frac{1}{12} \right) \\
&= \frac{1}{6}.
\end{aligned}
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"Success is the sum of small efforts repeated day-in and day-out." ~ Robert Collier