National University of Technology, Islamabad
Quiz VIII (Calculus II), Spring 2019
Solution Key

Q.1 Evaluate the line integral $\int_C (xy + z^3)ds$ from $(1, 0, 0)$ to $(-1, 0, \pi)$ along the helix C (see Figure 1) that is represented by the parametric equations

$$x(t) = \cos t, \quad y(t) = \sin t, \quad z = t, \qquad (0 \leq t \leq \pi).$$

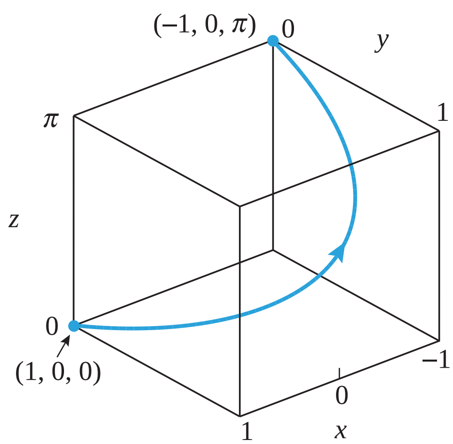

Figure 1: Helix C.

Sol. Note that $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$. Therefore, $\mathbf{v}(t) = -\sin t\mathbf{i} + \cos t\mathbf{j} + \mathbf{k}$ and $|\mathbf{v}(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{1+1} = \sqrt{2}$. Moreover, $f(x(t), y(t), z(t)) = x(t)y(t) - z^3(t) = \cos t \sin t + t^3$. Therefore, we have

$$\int_C (xy + z^3)ds = \int_0^\pi \left(x(t)y(t) - z^3(t)\right) |\mathbf{v}(t)|dt = \sqrt{2}\int_0^\pi (\cos t \sin t + t^3)dt$$

$$= \sqrt{2}\left[\frac{\sin^2 t}{2} + \frac{t^4}{4}\right]_0^\pi = \frac{\sqrt{2}\pi^4}{4}.$$

“Your problem isnt the problem, its your attitude about the problem.” — Ann Brashares.